St Joseph’s University
St Joseph’s Research Institute
School of Physical Sciences
Department of Mathematics
Paper Code-

Paper Title:

Unit-1 **10 hours**

Proof writing and techniques: Logic, Conditional and biconditional connectives, Quantifiers, Equivalences involving quantifiers, Proof strategies, Direct proof, Indirect proof, Contrapositive method, Proof by Contradiction, Proofs involving negations and conditionals, Proofs on quantifiers, Proofs involving conjunctions and biconditionals, Proofs involving disjunctions, Existence and uniqueness proofs, Mathematical Induction, Recursion, Strong Induction.

Unit-2 **10 hours**

Python Basics: Python syntax including variables, data types (int, float, string, list, tuple, dict, set), indexing and slicing, list comprehensions, conditional statements (if/elif/else), loops (for, while), function definition (def, lambda, map, filter), file I/O (reading/writing CSV using built-in csv module), and basic error handling (try/except).

NumPy & Matplotlib: Introduce essential sub-libraries: numpy.array for multidimensional arrays, numpy.linalg for linear algebra operations (dot, inv, eig, solve), numpy.random for random number generation, and numpy.linspace / numpy.meshgrid for grid creation. For plotting, use matplotlib.pyplot with sub function: plot (), scatter (), contour (), contour (), and imshow() for heatmaps.

Case Study

Unit-3 **40 hours**

Research Methodology for Applied Mathematics: Formulating a well-posed research question, conducting systematic literature reviews using databases (MathSciNet, arXiv, Scopus), and managing references with tools like Zotero or BibTeX. It covers mathematical modeling principles – translating real-world problems into differential equations, variational formulations – and the importance of non-dimensionalisation, scaling analysis, and asymptotic reasoning. To design numerical experiments: selecting appropriate discretization methods (finite difference, finite element, spectral), assessing convergence, stability (von Neumann analysis, CFL condition), and accuracy via verification (method of manufactured solutions) and validation against experimental or benchmark data. Handling of ill-posed problems, regularization techniques (Tikhonov, truncated SVD).

Research ethics: authorship, plagiarism, data fabrication, conflicts of interest, and ethical subjects where applicable. Writing and communication skills are emphasized – structuring a

mathematics paper (introduction, methods, results, discussion, appendix for proofs), effective use of LaTeX (cross-referencing, theorem environments, bibliography management), peer review process, responding to referee comments, and preparing conference presentations and posters.
Reproducibility and open science practices: sharing code and data via repositories (GitHub, Zenodo), writing reproducible Jupyter notebooks (as an option), and pre-registration of computational studies.
Assessment consists of a research proposal, a critical review of a published paper, and a short oral presentation on the student's intended thesis topic, evaluated on clarity, methodological soundness, and ethical awareness.

Recommended books

1. Daniel J. Velleman, *How to prove it: A structured Approach,* Cambridge University Press, 2009.
2. Richard Hammack, *Book of Proof*, Third edition, 2018.
3. David J. Pine**,** Introduction to Python for Science and Engineering (2nd Ed., 2025)

ST JOSEPH'S UNIVERSITY

ST. JOSEPH'S RESEARCH INSTITUTE

SCHOOL OF PHYSICAL SCIENCES: DEPARTMENT OF MATHEMATICS

Paper Code: ___________

Paper Title: ADVANCED FLUID MECHANICS AND THERMAL CONVECTION

Course Type: Ph.D. Coursework (Discipline Specific Course)

Teaching Hours: 60 Hours

Credits: 4

UNIT I: Concept of fluid as a deformable medium, classification of fluids, Newtonian and non-Newtonian fluid behaviour, shear stress and fluid deformation, fluid Properties and equation of state. Continuum Hypothesis, fluid as a continuous medium, Dimensions and Units: Fundamental and derived dimensions, dimensional homogeneity, dimensional analysis and dimensionless parameters in fluid mechanics. Mathematical Description of Fluid Motion. **20 hours**

UNIT II: Conservation Laws **-** Mass conservation, Linear momentum of conservation, angular momentum of conservation, Energy conservation. **10 hours**

UNIT III: Free convection, thermal convection, Rayleigh–Bénard convection and convective heat transfer. Boussinesq approximation, Free-free boundaries, Rigid boundaries, Critical Rayleigh number, Principle of exchange of stabilities, Porous medium, Dupit-Frochheier Relation, Darcy law, Brinkman's Model. **30 hours**

References

1. Chandrasekhar S., 2007, *Hydrodynamic and Hydromagnetic Stability*, Dover Publications (or Oxford University Press edition).
2. Kundu P. K., Cohen I. M. and Dowling D. R., 2016, *Fluid Mechanics*, 6th Edition, Academic Press.
3. Tritton D. J., 1979, *Physical Fluid Dynamics*, Van Nostrand Reinhold Company.
4. Turner J. S., 1973, *Buoyancy Effects in Fluids*, Cambridge University Press.
5. Nield D. A. and Bejan A., 2017, *Convection in Porous Media*, 5th Edition, Springer.

St Joseph's University
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School of Physical Sciences
Department of Mathematics
Paper Code-

Paper Title: Coloring of Graphs based on properties of domination and irredundance

Unit-1 **15 hours**

Graph Coloring, Domination, Matching, Factorization and Decomposition: Vertex Coloring, The Chromatic number of a graph, bounds for the Chromatic number, Color-Critical graphs, Perfect graphs, List colorings, Four Color Theorem, Matchings, 1-factors, Independence and covers, Domination, Factorization, Decomposition, Cycle Decomposition, Graceful Graphs

Unit-2 **15 hours**

Dominator Coloring and Global Dominator Coloring: Dominator coloring, the dominator chromatic number of a graphs, bounds, characterization of extremal graphs, global dominator coloring, the global dominator chromatic number of a graph, bounds and characterizations.

Unit-3 **15 hours**

Dominated Coloring and Global Dominated Coloring: Dominated coloring, the dominated chromatic number of a graphs, bounds, characterization of extremal graphs, global dominated coloring, the global dominated chromatic number of a graph, bounds and characterizations.

Unit-4 **15 hours**

Irredundance Coloring and Irredundance compelling Coloring: Compelling colorings, Irredundance coloring, the irredundance chromatic number of a graph, bounds, characterization of extremal graphs, irredundance compelling coloring, the irredundance compelling chromatic number, bounds and characterizations.

Recommended Books:

1. G. Chartrand, L. Lesniak and P. Zhang, Graphs and Digraphs, 6th Ed. (Chapman & Hall/CRC, New York, 2015).
2. T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, Fundamentals of Domination in Graphs. New York Academy of sciences, 1998.

Recommended Articles:

1. R. Gera, C. Rasmussen and S. Horton, Dominator colorings and safe clique parti tions, Congr. Numer. 181 (2006) 19–32.
2. R. M. Gera, "On dominator colorings in graphs," Graph Theory Notes of New York, vol. 52, no. 2, pp. 25–30, 2007.
3. I.S. Hamid and M. Rajeswari, Global dominator coloring of graphs, Discuss. Math. Graph Theory 39 (2019) 325–339.
4. M.A. Henning, Dominator and total dominator colorings in graphs, in: Structures of Domination in Graphs 66 Dev. Math., T.W. Haynes, S.T. Hedetniemi and M.A. Henning (Ed(s)) (Springer, Cham, 2021) 101–133.
5. S. Klavzar and M. Tavakoli, Dominated and dominator colorings over (edge) corona and hierarchical products, Appl. Math. Comput. 390 (2021) 125647.
6. H.B. Merouane, M. Haddad, M. Chellali and H. Kheddouci, Dominated coloring of graphs, Graphs Combin. 31 (2015) 713–727.
7. D.A. Kalarkop, I.S. Hamid, M. Chellali and P. Kaemawichanurat, Global Dominated coloring of graphs, Discussiones Mathematicae Graph Theory 44 (2024) 1293-1309.
8. R. Rangarajan and D. A. Kalarkop, "A note on global dominator coloring of graphs," Discrete Mathematics, Algorithms and Applications, vol. 14, no. 5, pp. 150–158, 2022.
9. D. Kalarkop, M. Henning, I. Sahul Hamid, and P. Kaemawichanurat, "On irredundance coloring and irredundance compelling coloring of graphs," Discrete Applied Mathematics, vol. 369, pp. 149–161, Nov. 2023.
10. D. A. Kalarkop and P. Kaemawichanurat, "Irredundance chromatic number and gamma chromatic number of trees," Communications in Combinatorics and Optimization, vol. 11, no. 1, pp. 269–275, 2024.